

# Non-equilibrium length in granular fluids: from experiment to fluctuating hydrodynamics

G. GRADENIGO, A. SARRACINO, D. VILLAMAINA AND A. PUGLISI

CNR-ISC and Dipartimento di Fisica, Università Sapienza - p.le A. Moro 2, 00185, Roma, Italy

PACS 45.70.-n – Granular systems

PACS 05.20.Dd – Kinetic theory

PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion

**Abstract.** - Velocity correlations in a quasi-2D driven granular fluid are studied in experiments and numerical simulations. The transverse velocity structure factor reveals two well defined energy scales, associated with the external “bath temperature”  $T_b$  and with the internal granular one,  $T_g < T_b$ , relevant at large and small wavelengths respectively. Experimental and numerical data are discussed within a fluctuating hydrodynamics model, which allows one to define and measure a non-equilibrium coherence length, growing with density, that characterizes order in the velocity field.

The study of collective phenomena in complex systems is a central issue in statistical mechanics. In particular, the degree of *order* emerging in such systems is customarily investigated by studying the behaviour of correlation functions [1]. However, in many systems which display interesting collective phenomena interactions are *non-conservative* and a stationary state is achieved through a continuous and homogeneous energy injection acting on every single particle: this is the case, for instance, of homogeneously vibrated granular materials as well as of the so-called “active fluids” including swarms of bacteria, birds, insects, fishes, pedestrians and even artificial motors [2–4]. In all those systems, the study of correlations requires tools beyond the standard equilibrium ones.

The emergence of order out of equilibrium is studied here in a granular system [5, 6], where inelastic collisions induce cross correlations between particles velocities. The behaviour of such correlations is customarily described with fluctuating hydrodynamics (FHD), which relies on a separation of time and length scales in the system, together with a suitable model for noise. Assessing the limits of validity of granular hydrodynamics is non-trivial [7, 8], but in several situations it has proven to be useful in explaining the behaviour of large scale correlations in granular fluids [9, 10]. In a more general context, the existence of non-trivial correlations emerging from non-equilibrium assumptions was already noticed in [11]. Recently, FHD also provided a prediction for the slowing-down of the dynamics, with increasing packing fraction, in granular flu-

ids [12, 13], a phenomenon which has a well known counterpart in elastic systems [14]. The above mentioned and similar results [15], which clearly show the existence of a time-scale growing with the density, suggest to look also for a growing length-scale.

In this paper we show the existence of a finite correlation length  $\xi$  in the velocity field, peculiar to a non-conservative fluid. Velocities correlations are investigated studying the velocity structure factor measured from experiments and numerical simulations. Data are discussed within a FHD model, allowing us to extract a length scale  $\xi$  and confirming that its associated order is a peculiarity of the out-of-equilibrium regime. The behaviour of  $\xi$  is investigated at different packing fractions, showing a remarkable increase of ordering in the velocity field with growing density. The large scale fluctuations of the velocity field suggest that a reference theoretical model where the thermostat has a finite viscous drag [16] is the most appropriate to modelize the experiments.

The experimental setup, sketched in Fig. 1, consists of an electrodynamic shaker (LDS V450) which vibrates vertically ( $\hat{z}$  axis) a horizontal rigid aluminium plate covered by a partial granular monolayer of  $N$  “steel 316” spheres, with packing fraction  $\phi = \pi n \sigma^2 / 4 < 0.5$ , where  $n = N/Area$  is the number density and the diameter  $\sigma$  of spheres is 4 mm. The plate is circular and has slowly rising boundaries in order to prevent highly dissipative head-on collisions between spheres and walls. The radius of the central flat region is  $R = 100$  mm. The surface of the plate



the existence of a finite correlation length in the velocity field. The oscillations of  $S_{\perp}(k)$  at high values of  $k$ , are likely to be a signature of an unexpected coupling with the density modes, already observed in previous works [21].

With the aim of giving a consistent interpretation of our experimental results, we now propose an effective model for the microscopic dynamics of particles (restricted to the plane  $xy$ ) and deduce from it a theoretical formula - through fluctuating hydrodynamics [21, 22] - for  $S_{\perp}(k)$ . Our proposal is well known in the literature [16]: it mimics the action of the rough vibrating plate by means of an interaction with an effective viscous “bath”. The model is described by the following equation governing the dynamics of the  $i$ -th particle ( $i \in [1, N]$ )

$$\dot{\mathbf{v}}_i(t) = -\gamma_b \mathbf{v}_i(t) + \boldsymbol{\zeta}_i(t) + \mathbf{F}_i, \quad (2)$$

where  $\mathbf{F}_i$  represents the effect of particle-particle inelastic hard core collisions with restitution coefficient  $\alpha$ , while  $\gamma_b^{-1}$  is the typical interaction time with the plate and  $\boldsymbol{\zeta}_i(t)$  a white noise, with zero mean and  $\langle \zeta_{i,\alpha}(t) \zeta_{j,\beta}(t') \rangle = 2T_b \gamma_b \delta_{\alpha\beta} \delta(t - t') \delta_{ij}$  ( $\alpha, \beta = \{x, y\}$ ). This kind of thermostat is well defined also in the elastic limit  $\alpha \rightarrow 1$ , where the fluid equilibrates to the bath temperature:  $T_g = T_b$  [16].

For the above model one can apply the usual kinetic theory to study hydrodynamic fields [21]. In linearized hydrodynamics, which applies to the moderately dense regime here studied, the transverse modes  $v_{\perp}(k, t)$  of the velocity field are decoupled from all the other modes. In particular, at small  $k$  they are well described - on average - by the equation  $\dot{v}_{\perp}(k, t) = -(\gamma_b + \nu k^2) v_{\perp}(k, t)$ , with  $\nu$  the kinematic viscosity [1]. This equation contains two main sources of relaxation: the external friction parametrized by  $\gamma_b$ , and vorticity diffusion parametrized by  $\nu$ . To analyze structure factors one needs to plug in appropriate noises. The two relaxation mechanisms described above are associated with two independent sources of randomness: external and internal noise, respectively. A rigorous evaluation of the way those noises enter the description for  $v_{\perp}(k, t)$  can be found in the literature for different models [22]. Here we present a more phenomenological approach, assessed by its ability to reproduce experimental and numerical results. The idea, partly inspired to previous granular FHD theories [21], consists in keeping valid - as a first approximation - the 2nd kind Fluctuation Dissipation Relation (FDR) for the two relaxation-noise pairs, separately. The result can be cast in a Langevin equation for the shear mode, with one single noise with appropriate variance:

$$\begin{aligned} \dot{v}_{\perp}(k, t) &= -(\gamma_b + \nu k^2) v_{\perp}(k, t) + \eta(k, t), \\ \langle \eta(k, t) \eta(-k, t') \rangle &= 2(\gamma_b T_b + \nu k^2 T_g) \delta(t - t'), \end{aligned} \quad (3)$$

with  $\langle \eta(k, t) \rangle = 0$ . The amplitudes of the external and internal noises in the above equation are reasonably assumed to be proportional to  $T_b$  and  $T_g$ , respectively. The p.d.f.

of  $v_{\perp}(k)$  measured in experiments are Gaussian, in agreement with Eq. (3), whereas those of real space velocities  $v_i$  show deviations from Gaussianity due to the relevance of collisions in Eq. (2). From Eq. (3) we immediately obtain

$$nS_{\perp}(k) = N^{-1} \langle |v_{\perp}(k)|^2 \rangle = \frac{\gamma_b T_b + \nu k^2 T_g}{\gamma_b + \nu k^2}. \quad (4)$$

At equilibrium, namely when collisions are elastic and  $T_g = T_b$ , equipartition between modes is perfectly fulfilled and the structure factor becomes flat, i.e.  $S_{\perp}(k) = T_b$ . Differently, in the granular case, where  $T_g \neq T_b$ , equipartition breaks down and from Eq. (4) we have that  $S_{\perp} \rightarrow T_b$  for small  $k$  and  $S_{\perp} \rightarrow T_g$  for large  $k$ . The Fourier inversion of Eq. (4) in two dimensions yields:

$$nG_{\perp}(\mathbf{r}) = T_g \delta^{(2)}(\mathbf{r}) + (T_b - T_g) \frac{K_0(r/\xi)}{\xi^2}, \quad (5)$$

where  $K_0(x)$  is the 2nd kind modified Bessel function that, for large distances, decays exponentially

$$K_0(r/\xi) \approx \sqrt{\frac{\pi}{2}} \frac{e^{-r/\xi}}{(r/\xi)^{1/2}}, \quad (6)$$

and we have introduced  $\xi = \sqrt{\nu/\gamma_b}$ . Such quantity turns out to be a *non-equilibrium* correlation length, measurable in the system *only* when non-conservative interactions are present. Indeed, at equilibrium, the second term in the r.h.s. of Eq. (5) vanishes and no coherence can be identified in the velocity field fluctuations.

The saturation of  $nS_{\perp}(k)$  to a finite value at small  $k$  provided from this theory is in agreement with the behaviour of experimental data observed in Fig. 2, suggesting that the model with finite friction is appropriate to describe the system.

The correlation length  $\xi$  is defined through  $\gamma_b$  and  $\nu$ , which can be obtained by studying the dynamical correlation  $C_{\perp}(k, t) = \langle v_{\perp}(k, t) v_{\perp}(-k, 0) \rangle / \langle |v_{\perp}(k, 0)|^2 \rangle$ . Indeed, from Eq. (3), we immediately obtain  $C_{\perp}(k, t) = \exp\{-(\gamma_b + \nu k^2)t\}$ . Such behaviour is in good agreement with experimental data, as reported in Fig. 3, where  $C_{\perp}(k, t)$  is plotted for several values of  $\phi$  (Fig 3(a)) and different wave-vectors  $k$  (Fig. 3(b)), together with the best fit curves. At each packing fraction, the length scale  $\xi = \sqrt{\nu/\gamma_b}$  can be obtained by comparing the decay of shear modes at different  $k$  (see Fig. 3(d)). A parabolic fit  $\tau^{-1}(k) = \gamma_b + \nu k^2$  shows that our data are broadly consistent with the theory, but it is also important to notice that only few data points are available. In Fig. 3(c) we see how the correlation length  $\xi$ , rescaled with the prediction - in the diluted approximation - for the mean free path  $\lambda_0 = 1/(2\sqrt{\pi}g_2n\sigma)$ , where  $g_2 = (1 - 7\phi/16)/(1 - \phi)^2$ , significantly grows with the packing fraction and signals an increasing degree of order in the granular fluid. Notice that the main contribution to the growth presented in Fig. 3(c) is due to the decrease of  $\lambda_0$ , while  $\xi$  grows only weakly, as already visible in Fig. 2, where  $S_{\perp}(k)$  has

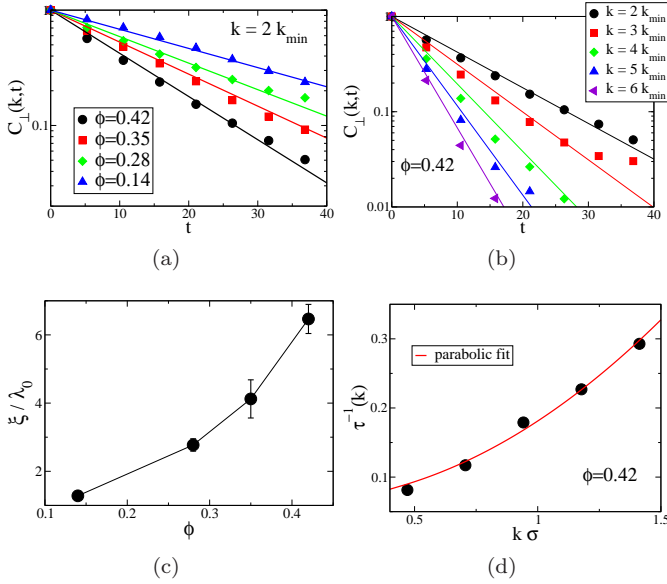


Fig. 3: (color online). Data from experiments. (a) Temporal decay of  $C_{\perp}(k, t)$  at fixed  $k = 2k_{\min}$  and (b) for different  $k$  at fixed  $\phi = 0.42$ . The unit of times is  $2\pi/\omega$ . (c) Correlation length  $\xi$ , rescaled by  $\lambda_0$ , plotted as function of  $\phi$ . (d) Parabolic fit of  $\tau^{-1}(k) = \gamma_b + \nu k^2$ .

a “sigmoidal” shape with an inflection point almost independent of  $\phi$ . Such an observation is consistent with the experimental measurements discussed in [17]. The growth of  $\xi/\lambda_0$  denotes an increasing number of correlated particles.

The finite extent of velocity correlations in a granular fluid was already pointed out in [17]. Here we discover that it is strictly related to a saturation of  $S_{\perp}(k)$  as  $k \rightarrow 0$ , corresponding to a well defined energy of the “external thermostat”. Within the homogeneous cooling regime a length-scale  $\xi$  was discussed in [10], but in that context it separated the stable,  $k > \xi^{-1}$ , from the unstable  $k < \xi^{-1}$ , modulations of the velocity field.

The above discussion shows that a fluctuation on the mode  $k$  decays with characteristic time  $\tau(k) = 1/(\gamma_b + \nu k^2)$ . This timescale has to be much larger than the microscopic time scale given by the mean collision time  $\tau_c \sim \lambda_0/\sqrt{T_g}$ , for the FHD with white noise to be effective. Comparing  $\tau(k)$  and  $\tau_c$ , we see that in our experiments such constraint is fulfilled for a narrow interval of small  $k$ . This range of validity is constrained by parameters such as the kinematic viscosity  $\nu$  and the drag  $\gamma_b$  of the thermal bath, which are not under control in experiments.

The predictions of the linearized FHD described above are tested by simulating an interacting system of inelastic hard disk modeled by Eq. (2), where  $\gamma_b$  enters as a fixed parameter. In particular,  $\gamma_b^{-1} \sim \tau_b$  is the timescale of the thermal bath, which can be tuned in order to have smaller values of  $T_g$ . Indeed, by lowering  $\gamma_b$ , also  $\nu$  is lowered, because, in first approximation,  $\nu \sim T_g^{1/2}$  [23]. Therefore, by tuning  $\gamma_b$ , we can raise the shear mode decay timescale

$\tau(k) = 1/(\gamma_b + \nu k^2)$ , extending the range of validity of FHD, i.e. the interval of  $k$  values where Eqs. (3) and (4) are expected to hold. We have performed 2D noisy event-driven molecular dynamics simulations of inelastic hard disks, at different packing fractions  $\phi \in [0.1, 0.5]$ , with  $\alpha \in [0.6, 1]$ . Details and results of the simulations are reported in a separate publication [24]. We consider  $N = 10000$  disks in a box of area  $L^2$  with periodic boundary conditions and the packing fraction is varied by changing the side length of the box. The particles have diameter  $\sigma = 0.01$  and unit mass and the thermal bath of Eq. (2) has parameters  $T_b = 1$  and  $\gamma_b = 1$ , while  $\tau_c$  is always in the range  $[0.005, 0.04]$ . In Fig. 4 we report our results for

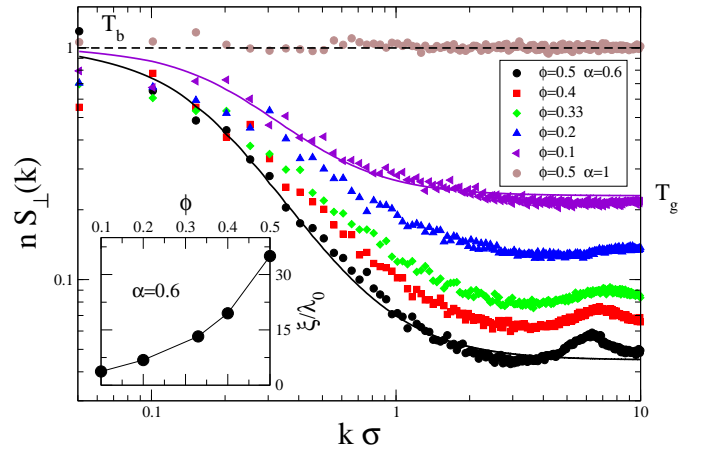


Fig. 4: (color online).  $S_{\perp}(k)$  measured in event driven molecular dynamics simulations, averaged over 400 frames. The solid lines are the best fit curves via Eq. (4), for  $\phi = 0.42$  and  $\phi = 0.14$ . Inset:  $\xi/\lambda_0$  as a function of  $\phi$ .

$S_{\perp}(k)$ . A striking analogy with the experimental results of Fig. 2 can be appreciated. Again, two energy scales are clearly observed: at small  $k$  all curves saturate to the value  $T_b = 1$ , in agreement with Eq. (4). Moreover, as expected, for  $\alpha = 1$  energy equipartition holds and  $S_{\perp}(k) = T_b$ , whereas for  $\alpha \neq 1$  the structure factor at large  $k$  oscillates around the granular temperature  $T_g$ . Notice that here, by using Eq. (4), we obtained a good fit of data, in a wide range of  $k$ , thus validating the FHD description (solid lines in Fig. 4). The kinematic viscosity  $\nu$  obtained from these fits, yields the correlation length  $\xi$ . A remarkable increase of  $\xi/\lambda_0$  as function of  $\phi$  is found (see the inset of Fig. (4)). Notice also that the gap between the granular temperatures at different packing fractions and the bath temperature is strongly increased, compared to the one observed in experiments. Such a large separation between the two energy scales is consistent with an extended realm of validity of the FHD.

In conclusion we have measured the transverse velocity structure factor, and its time decay, for a 2D driven granular fluid. We have observed a striking agreement with simulations of a microscopic model with stochastic driving and viscous friction, and with a fluctuating hydrodynam-



ics theory with two noise sources at different temperatures  $T_b \geq T_g$ . A central issue of our study has been to show the increasing extent of correlations in the velocity field, with growing density. Such a phenomenon emerges in the system only when time-reversal symmetry is broken, i.e. in the presence of an energy flux from the external driving to the system, due to the inequality  $T_g < T_b$ .

\* \* \*

We thank U.M.B. Marconi and A. Vulpiani for useful discussions and MD. Deen for help in building the experimental setup. The work is supported by the “Granular-Chaos” project, funded by the Italian MIUR under the FIRB-IDEAS grant number RBID08Z9JE.

## REFERENCES

- [1] FOSTER D., *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions* (Perseus Books) 1975.
- [2] JÜLICHER F., KRUSE K., PROST J. and JOANNY J.-F., *Phys. Rep.* , **449** (2007) 3.
- [3] NARAYAN V., RAMASWAMY S. and MENON N., *Science* , **317** (2007) 105.
- [4] CAVAGNA A. *et al.*, *Proc. Natl. Acad. Sci. USA* , **107** (2010) 11865.
- [5] JAEGER H. M. and NAGEL S. R., *Science* , **255** (1992) 1523.
- [6] DAUCHOT O., MARTY G. and BIROLI G., *Phys. Rev. Lett.* , **95** (2005) 265701.
- [7] KADANOFF L. P., *Rev. Mod. Phys.* , **71** (1999) 435.
- [8] GOLDBIRSCHE I., *Chaos* , **9** (1999) 659.
- [9] GOLDBIRSCHE I. and ZANETTI G., *Phys. Rev. Lett.* , **70** (1993) 1619.
- [10] VAN NOIJE T. P. C., ERNST M. H., BRITO R. and ORZA J. A. G., *Phys. Rev. Lett.* , **79** (1997) 411.
- [11] GRINSTEIN G., LEE D. H. and SACHDEV S., *Phys. Rev. Lett.* , **64** (1990) 1927.
- [12] TILL W., SPERL K. M. and ZIPPELIUS A., *Phys. Rev. Lett.* , **104** (2010) 225701.
- [13] VOLLMAYR-LEE K., ASPELMEIER T. and ZIPPELIUS A., *Phys. Rev. E* , **83** (2011) 001301.
- [14] BARRAT J. L., GOTZE W. and LATZ A., *J. Phys.: Condens. Matter* , **1** (1989) 7163.
- [15] SARRACINO A., VILLAMAINA D., GRADENIGO G. and PUGLISI A., *Europhys. Lett.* , **92** (2010) 34001.
- [16] PUGLISI A., LORETO V., MARCONI U. M. B., PETRI A. and VULPIANI A., *Phys. Rev. Lett.* , **81** (1998) 3848.
- [17] PREVOST A., EGOLF D. A. and URBACH J. S., *Phys. Rev. Lett.* , **89** (2002) 084301.
- [18] REIS P. M., INGALE R. A. and SHATTUCK M. D., *Phys. Rev. Lett.* , **98** (2007) 188301.
- [19] REIS P. M., INGALE R. A. and SHATTUCK M. D., *Phys. Rev. E* , **75** (2007) 051311.
- [20] BAUS M. and COLOT J. L., *J. Phys. C: Solid State Phys* , **19** (1986) L643.
- [21] VAN NOIJE T. P. C., ERNST M. H., TRIZAC E. and PAGONABARRAGA I., *Phys. Rev. E* , **59** (1999) 4326.
- [22] BREY J. J., MAYNAR P. and DE SORIA M. I. G., *Phys. Rev. E* , **79** (2009) 051305.
- [23] GARCÍA-ROJO R., LUDING S. and BREY J. J., *Phys. Rev. E* , **74** (2006) 061305.
- [24] GRADENIGO G., SARRACINO A., VILLAMAINA D. and PUGLISI A., *J. Stat. Mech.* , (2011) P08017.